

# CURRICULUM AND SYLLABI

*for*

## Minor Programme

(Applicable to 2022 admission onwards)



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राष्ट्रीय प्रौद्योगिकी संस्थान गोवा

NATIONAL INSTITUTE OF TECHNOLOGY GOA

कोट्टामल प्लेटू, कुंकोलिम, सालसेट, दक्षिण गोवा, गोवा - ४०३७०३, इंडिया

Kottamoll Plateau, Cuncolim, Salcete, South Goa, Goa- 403703, India

# Minor Specialization

in

## Computational Mathematics

*Offered by the*

## Department of Applied Sciences

Semester Offered	Course Code	Course Name	Type	L-T-P	Credits
IV	MA250M	Applied Mathematical Methods	MR	3-1-0	4
V	MA300M	Applications of Differential Equations	MR	3-1-0	4
VI	MA350M	Linear Programming	MR	2-1-0	3
VII	MA400M	Optimization	MR	2-1-0	3
VIII	MA450M	Advanced Numerical Methods	MR	3-1-0	4
<b>Total Credits</b>					<b>18</b>

## Detailed Syllabi of courses

Course Code	Course Name	L	T	P	Credits
MA250M	Applied Mathematical Methods	3	1	0	4

### Course Objective

This course is structured to provide engineers and scientists with a comprehensive grasp of advanced mathematical methods. It highlights crucial concepts such as Fourier integral and transform methods and calculus of variations, delving into their practical applications. Moving to the next level, the course introduces the mathematical modeling of engineering applications using ordinary differential equations (ODE) and difference equations. It is anticipated that students will develop a deep understanding of mathematical methods, modeling, and their real-world applications.

### Course Outcomes

At the completion of this course, the student shall acquire knowledge and ability

- CO1.** Acquire a knowledge of the basics of Fourier integral and Fourier transform methods, and subsequently apply these principles to tackle complex engineering problems
- CO2.** Understanding of the significance of calculus of variations and delve into its diverse applications across various fields
- CO3.** Develop a profound understanding of the significance of mathematical modeling through the ordinary differential equations (ODEs) and explore the extensive applications of ODEs in diverse fields.
- CO4.** Cultivate a deep comprehension of the importance of mathematical modeling utilizing difference equations, and investigate their broad applications across diverse fields.

## Relationship of Course Outcomes to Program Outcomes

H = High correlation; M = Medium correlation; L = Low correlation

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	H	H	H	M	M			L	L	M		H
CO2	H	H	H	H	H			L	L	M		H
CO3	H	H	H	H	H			L	L	M		H
CO4	H	H	H	H	H			L	L	M		H

### Syllabus

#### Module 1: Fourier Transform:

Fourier integral, Fourier cosine transforms, Fourier sine transforms, Fourier transform, Properties of Fourier transforms, Convolution for Fourier transforms, Parseval's identity for Fourier transforms, Discrete and fast Fourier transforms.

#### Module 2: Calculus of Variations:

Calculus of variations: Functional, variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema, Functional dependent on higher-order derivatives, Functional dependent on the function of several independent variables, Variational problems in parametric form, Sufficient condition for weak/storing extremum.

#### Module 3: Mathematical Modelling: ODE

Mathematical modelling through differential equations: Linear growth and decay models. Non-linear growth and decay models, compartment models. Mathematical modelling in population dynamics, mathematical models of epidemics, economics and medicine. Mathematical modeling of planetary motions, mathematical modeling of circular motion and motion of satellites, mathematical modeling through linear differential equations of the second order, miscellaneous mathematical models through ordinary differential equations of the second order.

#### **Module 4: Mathematical Modelling: Difference Equations:**

Mathematical modelling through difference equations: Basic theory of linear difference equations with constant coefficients, mathematical modelling through difference equation in economics, finance, population dynamics.

#### **Reference Books/Material**

1. E. Kreyszig, Advanced engineering mathematics (8th Edition), John Wiley 1999.
2. L. Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers, Moscow, 1970.
3. E. A. Bender, An Introduction to Mathematical Modeling, CRC Press, 2002.
4. M. M. Meerschaert, Mathematical Modeling, Elsevier Publ., 2007.
5. W. J. Meyer, Concepts of Mathematical Modeling, Dover Publ., 2000.

<b>Course Code</b>	<b>Course Name</b>	<b>L</b>	<b>T</b>	<b>P</b>	<b>Credits</b>
<b>MA300M</b>	<b>Applications of Differential Equations</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>4</b>

#### **Course Objective**

The objective of this course is to deliver students with a comprehensive understanding of stability analysis in autonomous systems, emphasizing critical points, stability types, and linear systems. Additionally, the course purposes to equip students with the knowledge of systems of differential equations, including existence theorems, homogeneous and non-homogeneous linear systems, and the application of eigenvalues and eigenvectors. Furthermore, the course intends to delve into boundary value problems, covering two-point problems, Green's functions, and their construction, while also addressing higher-order partial differential equations and their solution methods for constant coefficients.

#### **Course Outcomes**

At the completion of this course, the student shall acquire knowledge and ability

- CO1.** To analyze the stability of autonomous systems, identify critical points, and determine stability characteristics in linear systems.
- CO2.** Acquire the skills to solve and analyze systems of differential equations, both homogeneous and non-homogeneous, with a focus on linear systems featuring constant coefficients, eigenvalues, and eigenvectors.

**CO3.** Develop expertise in solving two-point boundary value problems and understanding the application of Green's functions, enabling them to handle non-homogeneous boundary conditions effectively.

**CO4.** Gain proficiency in solving equations of higher order and partial differential equations with constant coefficients.

### Relationship of Course Outcomes to Program Outcomes

**H = High correlation; M = Medium correlation; L = Low correlation**

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
<b>CO1</b>	H	H	H	M	M			L	L	M		H
<b>CO2</b>	H	H	H	H	H			L	L	M		H
<b>CO3</b>	H	H	H	H	H			L	L	M		H
<b>CO4</b>	H	H	H	H	H			L	L	M		H

### Syllabus

#### Module 1: Stability Analysis:

Autonomous systems. The phase plane and its phenomena, Types of critical points. Stability, Critical points and stability for linear systems.

#### Module 2: Systems of Differential Equations:

Existence theorems, homogeneous linear systems, non-homogeneous linear systems, linear systems with constant coefficients, eigenvalues and eigenvectors, diagonal and Jordan matrices.

#### Module 3: Boundary Value Problems:

Two-point boundary value problems, Green's functions, Construction of Greens functions, Non-homogeneous boundary conditions.

#### Module 4: Equations of Higher order-PDE:

Method of solution for the case of constant coefficients; Classification of second order equations; Reduction to canonical forms; Method of solution by separation of variables.

### Reference Books/Material

1. G.F. Simmons (1974), Differential Equations, TMH Edition, New Delhi.
2. E.A. Coddington (1999), An Introduction to Ordinary Differential Equations, PHI Learning.

3. U. Tyn Myint (1978), Ordinary Differential Equations, Elsevier North- Holland.
4. S. G. Deo and V. Raghavendra (2006), Ordinary differential equations, Tata McGraw Hill, New Delhi.
5. E.D. Rainville and P.E. Bedient (1969), Elementary Differential Equations, McGraw Hill, New York.
6. William E. Boyce and Richard C. DiPrima, " Elementary Differential Equations and Boundary Value Problems," 10TH Edition 2012 John Wiley & Sons, Inc.
7. A.C.King, J.Billingham and S.R.Otto (2006), Differential equations, Cambridge University Press.
8. S.L. Ross (1984), Differential equations, 3rd edition, John Wiley & Sons, New York.

Course Code	Course Name	L	T	P	Credits
MA350M	Linear Programming	2	1	0	3

### Course Objective

This course is structured to provide engineers with a comprehensive grasp of advanced concepts of Operations Research (OR) which is a discipline that helps to make better decisions in complex scenarios by the application of a set of advanced analytical methods. It couples theories, results and theorems of mathematics, statistics and probability with its own theories and algorithms for problem solving. Applications of these techniques spread over various fields in engineering, management and public systems.

### Course Outcomes

At the completion of this course, the student shall acquire knowledge and ability

- CO1.** Acquire a knowledge of the basics of LPP, and subsequently apply these principles to tackle complex engineering problems
- CO2.** Understanding of the significance different algorithms to solve LPP and delve into its diverse applications
- CO3.** Develop a profound understanding of the significance of mathematical methods in solving LPP
- CO4.** Cultivate a deep comprehension algorithms in Operations research in solving real world problems like transportation problems and others.

## Relationship of Course Outcomes to Program Outcomes

H = High correlation; M = Medium correlation; L = Low correlation

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	H	H	H	M	M			L	L	M		H
CO2	H	H	H	H	H			L	L	M		H
CO3	H	H	H	H	H			L	L	M		H
CO4	H	H	H	H	H			L	L	M		H

## Syllabus

### Module 1: Linear Programming:

Linear Programming: Introduction to linear algebra. Solution of a system of linear equations, Linear independence and dependence of vectors, Concept of basis, Basic feasible solution, Convex sets. Extreme points, Hyperplanes and Halfspaces, Convex cones, Polyhedral sets and cones.

### Module 2: Solution of Linear LPP :

Linear Programming Problem Formulation, solution by Graphical method, Theory of simplex method, Simplex algorithm, Two phase method, Charnes-M method, Degeneracy, Theory of duality, Dual-simplex method. Revised simplex method, Bounded variable linear programming problem, Interior point algorithm for linear programming problem. Introduction to linear integer programming.

### Module 3: Applications of Linear LPP:

Transportation problem (TP) and its formulation. finding basic feasible solution of TP using North-West Corner Rule, Least Cost and Vogel's Approximation Method, MODI method for finding optimal solution for TP, Assignment problem and its formulation, Hungarian method for solving Assignment problem, Transshipment and Travelling salesmen problem.

### Reference Books/Material

1. Luenberger D G, Introduction to Linear and Nonlinear Programming, Addison Wesley, 1984.
2. G. Hadley: Linear Programming. Narosa, Reprint, 2002.
3. Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.
4. E. Kreyszig, Advanced engineering mathematics (8th Edition), John Wiley (1999).



Course Code	Course Name	L	T	P	Credits
MA400M	Optimization	2	1	0	3

### Course Objective

Optimization holds an important place in both practical and theoretical worlds, as understanding the timing and magnitude of actions to be carried out helps achieve a goal in the best possible way. This course emphasizes data-driven modeling, theory and numerical algorithms for optimization with real variables. Explore the study of maximization and minimization of mathematical functions and the role of prices, duality, optimality conditions, and algorithms in finding and recognizing solutions. Learn about applications in machine learning, operations, marketing, finance and economics.

### Course Outcomes

At the completion of this course, the student shall acquire knowledge and ability

**CO1.** Acquire a knowledge of the basics of non-linear optimization problems and apply these principles to solve problems of real-life situation

**CO2.** Understanding of the significance of numerical algorithms and delve into its diverse variations in them

**CO3.** Develop a understanding of the significance of mathematics behind the numerical algorithms by coding and applying them

### Relationship of Course Outcomes to Program Outcomes

**H = High correlation; M = Medium correlation; L = Low correlation**

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
<b>CO1</b>	H	H	H	M	M			L	L	M		H
<b>CO2</b>	H	H	H	H	H			L	L	M		H
<b>CO3</b>	H	H	H	H	H			L	L	M		H
<b>CO4</b>	H	H	H	H	H			L	L	M		H

## Syllabus

### Module 1: Unconstrained Optimization:

Basic Properties of Solutions and Algorithms, First-Order Necessary Conditions, Second-Order Conditions, Convex and Concave Functions, Minimization and Maximization of Convex Functions; Basic Descent Methods-Fibonacci and Golden Section Search, Line Search by Curve Fitting, Newton's Method.

### Module 2: Conjugate Direction Methods:

Conjugate directions, The conjugate gradient method, Quasi-Newton methods- Modified Newton method, Davidon-Fletcher-Powell method, The Broyden family, Combination of steepest descent and Newton's method. Solving  $Ax=b$ .

### Module 3: Constrained Minimization :

First-order necessary conditions (Equality Constraints), Second-order conditions(KKT-conditions), and Lagrange multipliers; The gradient projection method, Penalty and barrier methods, Primal-dual methods, The standard problem.

### Reference Books/Material

1. E. Kreyszig, Advanced engineering mathematics (8th Edition), John Wiley (1999)
2. Luenberger D G, Introduction to Linear and Nonlinear Programming, Addison Wesley, 1984.
3. Fletcher R, Practical methods of Optimization John Wiley, 1980.
4. Edwin Chong, Stanislaw Zak, An Introduction to Optimization, Wiley Student Edition.
5. Sheldon M. Ross, "Introduction to Probability Models", 11th Edition, Academic Press.

Course Code	Course Name	L	T	P	Credits
MA450M	Advanced Numerical Methods	3	1	0	4

### Course Objective

This course is designed to equip engineers and scientists with a robust understanding of advanced numerical methods. Emphasizing essential concepts like finite difference methods and partial differential equations, students will cultivate a profound knowledge of applied mathematics and its practical applications

## Course Outcomes

At the completion of this course, the student shall acquire knowledge and ability

**CO1.** Attain a firm understanding of fundamental finite difference methods and apply them to address challenging engineering problems.

**CO2.** Comprehend the importance of elliptic differential equations, explore their applications, and learning the techniques for solving them through finite difference methods.

**CO3.** Comprehend the importance of parabolic differential equations, explore their applications, and learning the techniques for solving them through finite difference methods.

**CO4.** Comprehend the importance of hyperbolic differential equations, explore their applications, and learning the techniques for solving them through finite difference methods.

## Relationship of Course Outcomes to Program Outcomes

**H = High correlation; M = Medium correlation; L = Low correlation**

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
<b>CO1</b>	H	H	H	M	M			L	L	M		H
<b>CO2</b>	H	H	H	H	H			L	L	M		H
<b>CO3</b>	H	H	H	H	H			L	L	M		H
<b>CO4</b>	H	H	H	H	H			L	L	M		H

## Syllabus

### Module 1: Introduction to Finite Difference Methods:

Finite difference approximations to derivatives, notations, finite difference method, linear problem with Dirichlet and non-Dirichlet boundary conditions, nonlinear problems.

### Module 2: Elliptic Partial Differential Equations:

Elliptic partial differential equations: Poisson equation on a rectangular domain, Dirichlet boundary conditions, non-Dirichlet boundary conditions, solving the discrete equation, relaxation methods, convergence analysis.

**Module 3: Parabolic Partial Differential Equations:**

The heat equation with Dirichlet boundary conditions, forward, backward and Crank-Nicolson method, absolute stability. General parabolic equations, non-Dirichlet boundary conditions, stability analysis, problems in two spatial domains.

**Module 4: Hyperbolic Partial Differential Equations:**

Advection equation, upwind differencing, stability analysis, MacCormack method, the wave equation, stability analysis.

**Reference Books/Material**

1. R. L. Burden and J. D. Faires, Numerical Analysis, Ninth Edition, Cengage Learning, 2011.
2. B. Bradie, A friendly introduction to numerical analysis, Pearson Education, 2007.
3. G. D. Smith, Numerical Solution of P.D.E., Oxford University Press, New York, 1995.
4. C. F. Gerald and P. O. Whestley, Applied Numerical Analysis, Seventh Edition, Pearson Education, 2008.